A Mathematical Model for Distributed Channel Access in Cognitive Radio Networks

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06.07.2011

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MOTIVATION

- **Need for cognitive radio (CR) solutions**
  - For QoS restricted applications
    - Voice, video, etc.
  - Also for low priority applications
    - Health control (average heart beat), road congestion control (central tracking system), door opener, etc.
    - Can tolerate >100 ms delay

MOTIVATION

- **Most of the CR MAC protocols have no mathematical model**
  - Hard to trust simulation results
- **Problems in implementing proposed solutions**
  - Modify existing devices, protect primary service quality, expensive hardware, etc.
- **No efficient CR access solutions for digital data channels (WiFi)**

MOTIVATION

- **The aim of this study**
  - Propose a MAC protocol for SUs
    - Grants effective solutions for SU access
    - Utilize white spaces in WiFi channels
    - Limit harm against PUs
  - Adapt 802.11e mathematical model for secondary access
  - Determine optimal parameter set for SU access to create strict prioritization

802.11E DIFFERENTIATION

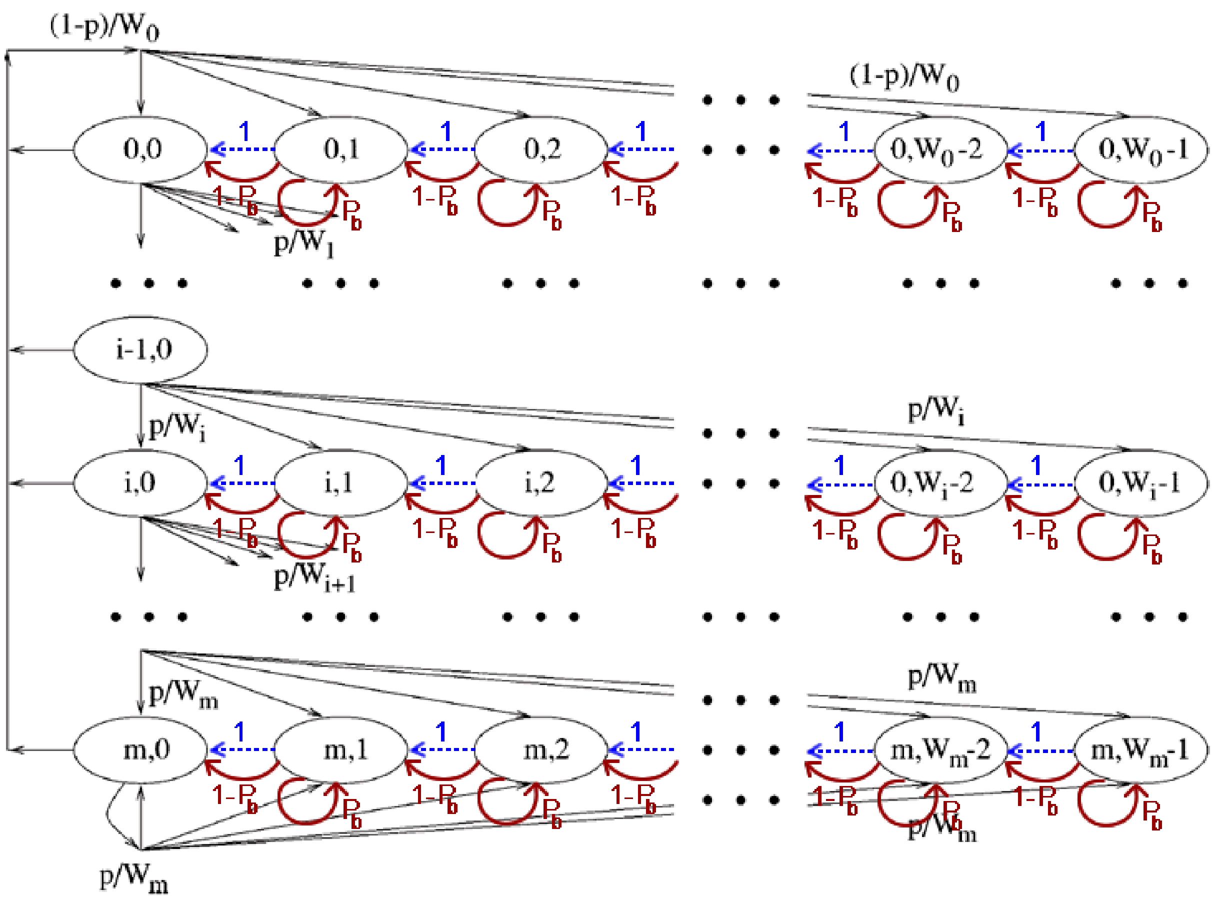
- **IEEE 802.11e provides service differentiation**
  - Enhanced Distributed Coordination Function (EDCA)
- **Min and Max contention window size**
  - $CW_{min}^{(i)}$ and $CW_{max}^{(i)}$ for service class i
  - Choose a uniform random parameter R from its interval $[0, CW_j - 1]$

802.11E PARAMETER SET

- **Arbitration inter-frame space ($AIFS^{(i)}$)**
  - Before entering the backing off stage
  - Each class i station has to wait for $AIFS^{(i)}$ duration and sense the channel as idle
- **Transmission opportunity ($TXOP^{(i)}$)**
  - Permits consecutive frame transmissions for the winner station
  - Contention grants $TXOP^{(i)}$ transmission duration

802.11E ANALYTICAL MODEL

- Discrete Markov Model of 802.11



[Bianchi00]

[Ziouva02]

Correcting by adding busy period

802.11E ANALYTICAL MODEL

- **$b_{i,j,k}$: stationary distribution of each node for the Markov chain**
  - i denotes the priority class
    - Note: Similar for 802.11e [Robinson04]
    - N_i: Number of classes
  - j stands for the backoff stage
  - k is the backoff counter
- **c_i^j: collision probability**

802.11E
ANALYTICAL MODEL

- **Set of equations for saturation conditions**

$$b_{i,j,0} = c_i^j b_{i,0,0} \quad (1)$$

$$b_{i,m_i,0} = \frac{c_i^{m_i} b_{i,0,0}}{1 - c_i} \quad (2)$$

$$b_{i,j,k} = \frac{CW^{(i,j)} - k}{CW^{(i,j)}} \frac{1}{1 - p_b} b_{i,j,0} \quad (3)$$

$$b_{i,0,0} = \frac{1 - (1 - c_i) - (1 - p_b)}{1 - p_b} b_{i,-1,0} \quad (4)$$

$$b_{i,-1,0} + \sum_{j=0}^{m_i} \sum_{k=0}^{CW^{(i,j)}-1} b_{i,j,k} = 1 \quad (5)$$

802.11E ANALYTICAL MODEL

- **τ_i: attempt probability of a class i station**

$$\tau_i = \sum_{j=0}^{m_j} b_{i,j,0} = \frac{2(1-p_b)(1-2c_i)}{\alpha\beta+\gamma} \qquad (6)$$

- **where**

$$p_b = 1 - \prod_{h=1}^{N} (1-\tau_h)^{N_h} \qquad (7)$$

$$\alpha = 1-(1-c_i)(1-p_b) \qquad (8)$$

$$\beta = 1-2c_i+(1-c_i-2^{m_i}{c_i}^{m_i+1})CW^{(i,0)} \qquad (9)$$

$$\gamma = 2(1-c_i)(1-2c_i)(1-p_b)^2. \qquad (10)$$

802.11E ANALYTICAL MODEL

- c_i: **collision prob. of a class i station**

$$c_i = 1 - (1-\tau_i)^{N_i - 1} \prod_{j \neq i} (1-\tau_j)^{N_j} \qquad (11)$$

- **We have (K: number of classes)**
  - $\tau_i = \mathrm{f}(c_i)$
  - $c_i = \mathrm{g}(\tau_i)$
- $2K$ **unknowns in these $2K$ equations**
  - τ_i and c_i can be derived numerically

$$\tau_i = \frac{2(1-p_b)(1-2c_i)}{\alpha\beta+\gamma} \qquad (6)$$

802.11E ANALYTICAL MODEL

- **Until now there is no AIFS**
  - How to add $AIFS^{(i)}$?
- **Divide the channel activity into K different access zones** [Robinson04]

$$AIFS_1 < AIFS_2 < \cdots < AIFS_K$$

- **ACs with indexes $j \leq i$ are allowed to contend in zone i**
- **π_i: Stationary probability that the system is in zone i**

802.11E ANALYTICAL MODEL

- **We can edit $c_i = \mathrm{g}(\tau_i)$ to $c_i{}' = \mathrm{g}'(\tau_i)$ as**

$$c_i{}' = \sum_{j=i}^{K} \left[\frac{\pi_j}{\sum_{k=j}^{K} \pi_k} \left(1 - \frac{1}{1-\tau_i} \prod_{k=1}^{j} (1-\tau_k)^{N_k} \right) \right] \qquad (12)$$

- **γ_i: normalized throughput of a class i station (renewal theory)**

$$\gamma_i = \frac{E[payload\ transmitted\ in\ a\ cycle]}{E[length\ of\ a\ transmission\ \ cycle]} = \frac{P_{succ}^{(i)} L_{packet}}{L_{cycle}} \qquad (13)$$

- **Model is very accurate for small $AIFS$ ($AIFS \ll CW$) and larger N_i** [Kumar08]

COGNITIVE RADIO MODEL

- **Assume that we have two classes**
  - Primary access category (PAC)
  - Secondary access category (SAC)

$$\begin{aligned} c_{PU} &= \pi_{PU}[1-(1-\tau_{PU})^{N_{PU}-1}] \\ &\quad +\pi_{ALL}[1-(1-\tau_{PU})^{N_{PU}-1}(1-\tau_{SU})^{N_{SU}}] \end{aligned} \tag{14}$$

$$c_{SU} = 1-(1-\tau_{PU})^{N_{PU}}(1-\tau_{SU})^{N_{SU}-1} \tag{15}$$

  - π_{PU}: probability that only PUs are contending for spectrum resources
  - $\pi_{ALL} = 1 - \pi_{PU}$

COGNITIVE RADIO MODEL

- **q_{PU}: the probability that the slot is empty in primary zone**
  - q_{ALL} the probability that the slot is empty in second zone

$$q_{PU} = \sum_{j=1}^{N_{PU}} (1 - c_{PU}) \qquad (16)$$

$$q_{ALL} = \prod_{j=1}^{N_{PU}} (1 - c_{PU}) \prod_{k=1}^{N_{SU}} (1 - c_{SU}) . \qquad (17)$$

COGNITIVE RADIO MODEL

- **Interframe spacing difference between PU and SU is $l = AIFS_{SU} - AIFS_{PU}$**
- **Linear Markov chain approximation, $\pi_{(i)}$ can be calculated as**

$$\pi_{PU} = \frac{1 + q_{PU} + q_{PU}^2 + \cdots + q_{PU}^{l-1}}{1 + q_{PU} + q_{PU}^2 + \cdots + q_{PU}^{l-1} + q_{PU}^l/(1 - q_{ALL})} \quad (18)$$

$$\pi_{ALL} = \frac{q_{PU}^l/(1 - q_{ALL})}{1 + q_{PU} + q_{PU}^2 + \cdots + q_{PU}^{l-1} + q_{PU}^l/(1 - q_{ALL})}. \quad (19)$$

COGNITIVE RADIO MODEL

- **Using attempt and collision probabilities**
  - Throughput

$$\gamma_d = \frac{\pi_{ALL}\sigma_d N_{SU}\tau_{SU}(1-\tau_{SU})^{N_{SU}-1}(1-\tau_{PU})^{N_{PU}}}{\pi_{PU}E[S_{PU}]+\pi_{ALL}E[S_{ALL}]} \quad (20)$$

  - Access Delay

$$A_D^{(i)} = E\left[S^{(i)}\right]\left[\frac{CW_{min}^{(i)}}{2}\left(\frac{1-(2c_i)^{m_i}}{1-2c_i}+\frac{(2c_i)^{m_i}}{1-c_i}\right)\right] + \frac{c_i}{1-c_i}E\left[T_{coll}^{(i)}\right]+T_{succ}^{(i)} \quad (21)$$

COGNITIVE RADIO PARAMETER SELECTION

- **Objectives**
  - Strict prioritization between PUs and SUs
    - $AIFS^{(SU)}$ parameter
  - Maximizing the throughput for SUs
    - $CW_{min}^{(SU)}$ and $CW_{max}^{(SU)}$ parameters
  - Supplying fairness between SUs
    - CSMA based design

COGNITIVE RADIO PARAMETER SELECTION

- **Desicions**
  - $AIFS_{SU} = AIFS_{PU} + CW_{min}^{(\mathrm{PU})}$ for strict prioritization
    - Need PU-PU collision for extra harm
  - Need adaptive CW for SUs to increase throughput (according to number of SUs)
  - To limit harm against QoS of PUs
    - $TXOP^{(SU)}$ equals one packet transmission

SIMULATION RESULTS

- Default EDCA parameter set for each AC

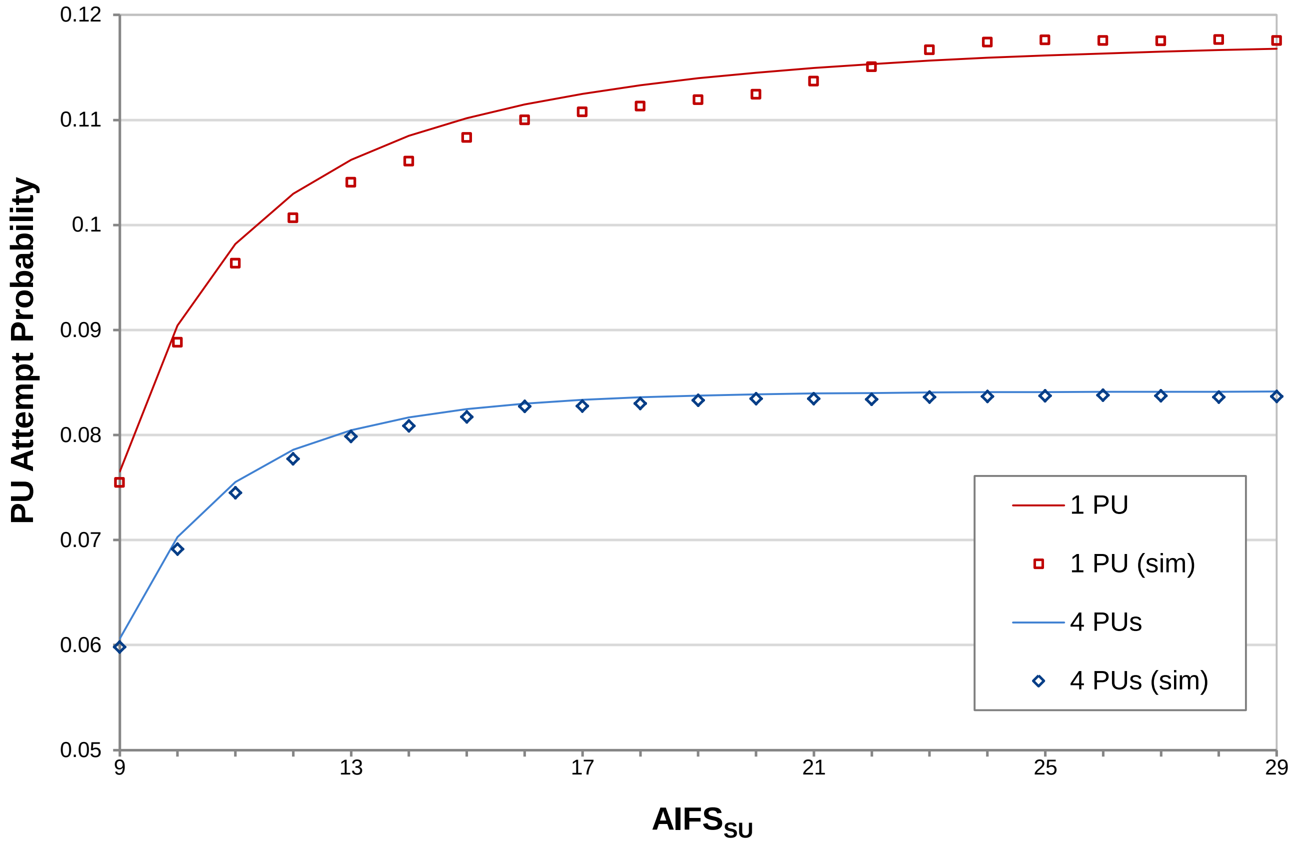
AC	CW_{min}	CW_{max}	AIFS
AC_BK (background)	16	1024	79μs
AC_BE (best effort)	16	1024	43μs
AC_VI (voice)	8	16	34μs
AC_VO (video)	4	8	34μs

- Group all ACs into primary AC (PAC) for simplicity
- 4 SUs
- Single channel!

Parameter	Value
$CW_{max}^{(PU)} = CW_{max}^{(SU)}$	1024 slots (≈ 9ms)
$CW_{min}^{(PU)} = CW_{min}^{(SU)}$	16 slots (144μs)
$m^{(PU)} = m^{(SU)}$	6
$AIFS_{PU}$	**9 slots (81μs)**
$AIFS_{SU}$	**25 slots (225μs)**
$TXOP_{PU} = TXOP_{SU}$	10 slots (1 packet ≈ 640 byte)
Channel Bandwidth	54Mbit/s
Slot Duration	9μs

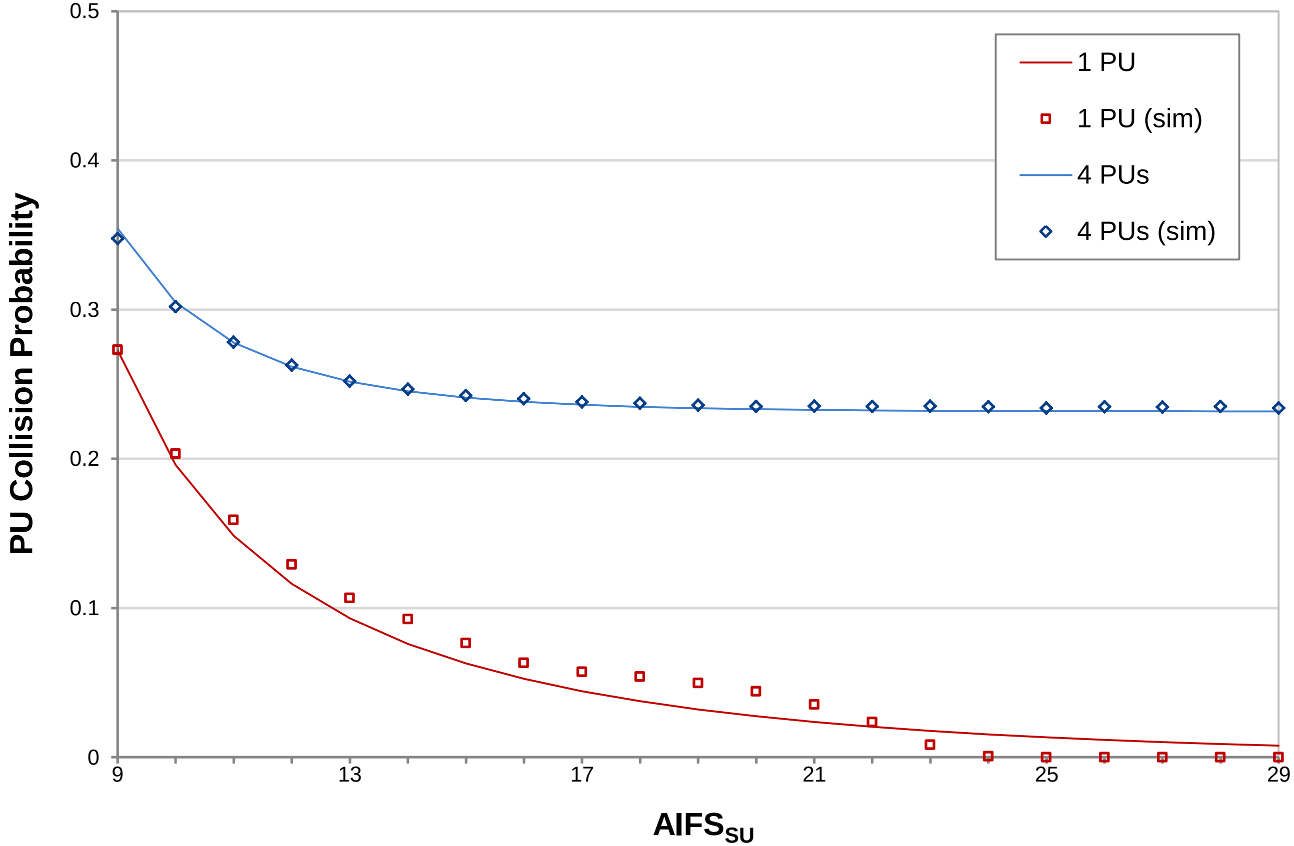
SIMULATION RESULTS

- PU Attempt Probability - $AIFS_{SU}$



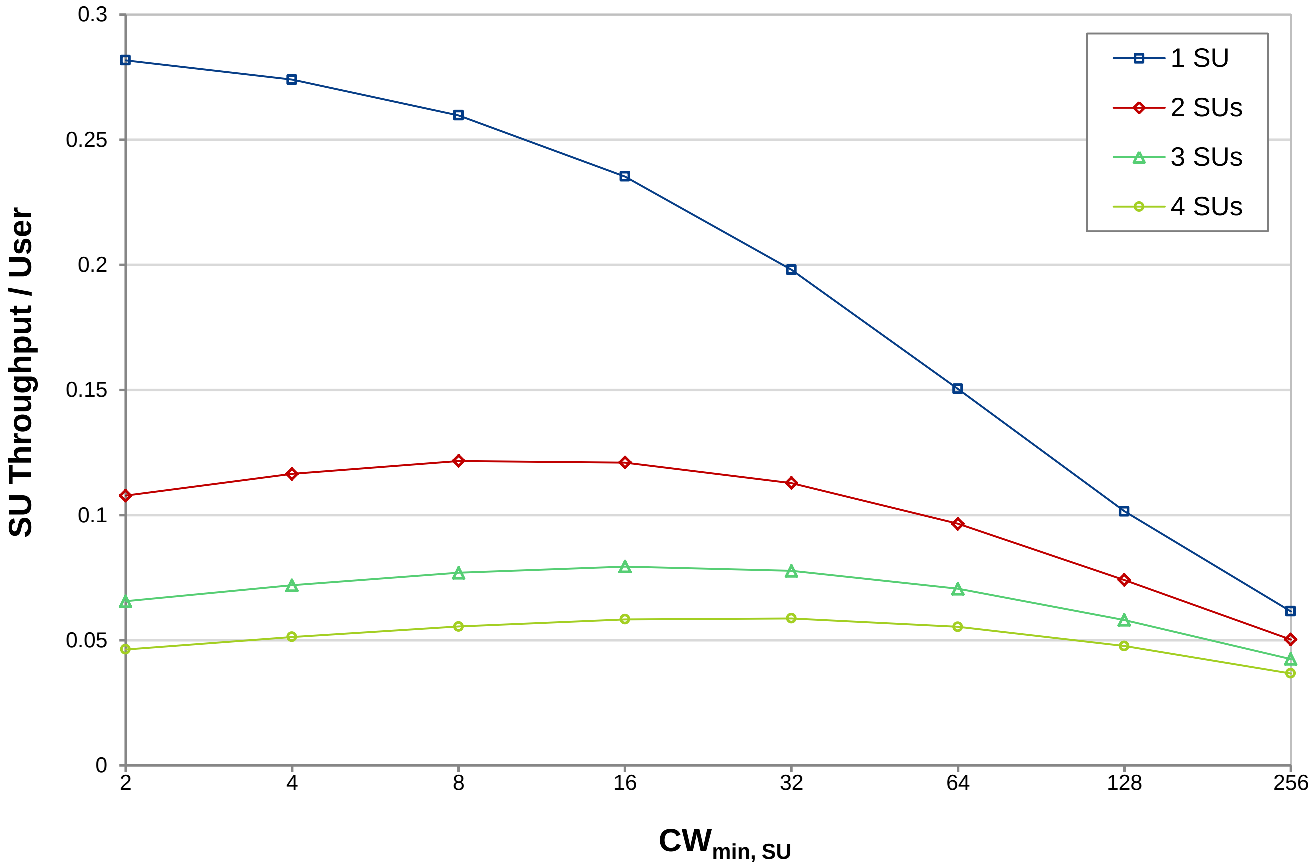
SIMULATION RESULTS

- PU Collision Probability - $AIFS_{SU}$



SIMULATION RESULTS

- SU Throughput / User – $CW_{min,SU}$



CONCLUSION

- **Adapt 802.11e priority model for CRs**
  - Supply access for SUs by limiting harm against service quality of PUs
- **Create strict prioritization between PUs and SUs**
  - Modifying EDCA parameter set
- **The proposed solution is applicaple**
  - Simulation and mathematical results

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